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Theoretical and Experimental Study of Falling- Cylinder Rheometer

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Abstract

The present study attempted deriving and solving the falling- cylinder governing equations for power law model non-Newtonian fluids. Based on this theoretical study, a novel falling- cylinder rheometer (FCR) was designed and constructed to measure the rheological properties of non-Newtonian fluids. Different falling cylinders with predesigned densities were used to determine the apparent viscosity of polyvinyl alcohol (PVA) solutions in water with various concentrations. The results indicate that all PVA solutions obey the power law model with the power law index as well as the consistency index changing linearly with concentration. Increasing concentration of the solution decreases power law index, while enhances consistency index and apparent viscosity.

Keywords: *Falling-Cylinder Rheometer, Non-Newtonian, Polyvinyl alcohol, Power Law, Falling object*

Introduction

The study of flow past a rigid object is of great practical importance and because it is the foundation of a branch of fluid mechanics, namely particle mechanics, it has been a subject of many theoretical, numerical and experimental investigations. An extensive literature review can be found in Happel and Brenner [1], which deals with particle motions in a Newtonian fluid at low Reynolds numbers.

Particle mechanics is the theoretical basis of an experimental approach known as the falling- object viscometry, which consists of allowing an object to fall freely in a fluid. After initial acceleration, when the external drag on the surface and buoyancy become equal to the downward force due to gravity, the object attains a constant terminal velocity. Because of their fast and simple operation, falling balls have been widely used to determine the viscosity of the Newtonian and non-Newtonian fluids [2-5]. The falling cylinder viscometers are preferred to falling-ball viscometers because it is easier to construct a cylinder with a predesigned density and it is more amenable to analysis of non-Newtonian fluids than a spherical ball. Design and evaluation of the falling object rheometers for non-Newtonian fluids were performed by Park and Irvine [6] and Park [7]. Their mathematical investigations were conducted by assuming no end effects, in essence, a cylinder of infinite length. The end effects of Newtonian fluids were considered by several investigators [8-13]. Etemad, *et al.* [14] obtained the falling equations for a steady, axisymmetric flow of an income-

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pressible, power-law model non-Newtonian fluid with egligible convective terms.

The purpose of the present article is to obtain an appropriate solution for falling-cylinder rheometer for power- law fluids, and then design and construction of an FCR to measure the rheological properties of polyvinyl alcohol with different concentrations.

Theory

When a cylinder is dropped in a viscous fluid, after initial acceleration, the cylinder attains a constant velocity known as terminal velocity. There are three forces acting on the cylinder: force of gravity (acting downward), buoyancy force (acting upward), and drag force (acting upward). It is assumed that the ratio of the length to the radius of the cylinder (L/R) is sufficiently large for the end effects to be negligible. When terminal velocity is reached, the Newton's second law reduces to the requirement that the forces on the cylinder algebraically sum up to zero.

Figure 1 presents the falling- cylinder rheometer.

The power law model constitutive equation is as follows:

$$\tau_{rz} = -m\left(\frac{\partial V_z}{\partial r}\right)^n \quad K \le R_o \le X \tag{1}$$

$$\tau_{rz} = m\left(\frac{-\partial V_z}{\partial r}\right)^n \quad X \le R_o \le 1 \tag{2}$$

where R_o and X are dimensionless parameters and are defined as $R_o = \frac{r}{R_t}$ and $X = \frac{R_m}{R_t}$ and R_m is the radius at which maximum velocity occurs.

The equation of motion in z-direction and boundary conditions are simplified to:

$$\frac{\partial (r\tau_{rz})}{\partial r} = \frac{r\Delta P}{L} \tag{3}$$

where: $\frac{\Delta P}{L} = \frac{-dp}{dz} + \rho g$

$$\tau_{rz} = 0 \text{ at } r = XR_t \tag{4}$$

Integration of (3) with the boundary condition (4) gives:

$$\tau_{rz} = R_t \Delta P\left(R_o - \frac{X^2}{R_o}\right)/2L \tag{5}$$

Combination of equations (1), (2) and (5) results in:

$$\frac{\partial V_z}{\partial r} = \left(\frac{R_t \Delta P}{2mL}\right)^{1/n}\left(\frac{X^2}{R_o} - R_o\right)^{1/n} \quad K \le R_o \le X \tag{6}$$

$$\frac{-\partial \mathrm{V}_z}{\partial \mathrm{r}} = \left(\frac{R_t \Delta P}{2mL}\right)^{1/n}\left(R_o - \frac{X^2}{R_o}\right)^{1/n} \quad X \le R_o \le 1 \tag{7}$$

The dimensionless velocity (U_z) is defined as follows:

$$U_z = \frac{Vz}{[R_t(R_t \Delta P/2mL)^{1/n}]} \tag{8}$$

Equations (6) and (7) may be written as:

$$\frac{\partial U_z}{\partial R_o} = \left(\frac{X^2}{R_o} - R_o\right)^{1/n} \tag{9}$$

$$\frac{\partial U_z}{\partial R_o} = -\left(R_0 - \frac{X^2}{R_o}\right)^{1/n} \quad X \le R_o \le 1 \tag{10}$$

Force balance over the cylinder gives:

$$\pi K^2 R_t^2 \Delta P + \pi K^2 R_t^2 g \rho_l L$$

$$+ 2\pi K R_t L(-\tau_w) = \pi K^2 R_t^2 g \rho_s L \tag{11}$$

Combination of equations (5) and (11) on cylinder wall $(R_o = K)$ yields:

$$\frac{\Delta P}{L} = \left(\frac{K}{X}\right)^2 g(\rho_s - \rho_l) \tag{12}$$

Substitution of equation (12) into (5) for $(R_o = K)$ gives:

$$\tau_w = \left(\frac{K}{X}\right)^2\left(K - \frac{X^2}{L}\right) g R_t(\rho_s - \rho_l)/2 \tag{13}$$

A mass balance related to displaced fluid by moving cylinder to fluid flowing between cylinder and tube gap becomes as follows:

$$2\pi R_t^2 \int_K^1 V_z R_o dR_o = \pi K^2 R_t^2 V_t \tag{14}$$

With respect to dimensionless velocity definition:

$$\frac{KU_t}{2} = \int_K^1 U_z R_o dR_o \tag{15}$$

The integration of equation (15) by parts results in:

$$\int_K^1 (\frac{dU_z}{dR_o}) R_o^2 dR_o = 0 \tag{16}$$

Combination of equations (9), (10), and (16) yields:

$$\int_K^X (\frac{X^2}{R_0} - R_o)^{1/n} R_o^2 dR_o$$

$$-\int_X^1 (R_o - \frac{X^2}{R_o})^{1/n} R_o dR_o = 0 \tag{17}$$

If values of n and K are specified, the value of X can be calculated from (17). The integration of (9), and (10) with the following boundary conditions gives:

$$U_z = -U_t \quad \text{at} \quad R_o = K \tag{18}$$

$$U_z = 0 \quad \text{at} \quad R_o = 1 \tag{19}$$

$$U_z + U_t = \int_K^{R_o} (\frac{X^2}{R_o} - R_o)^{1/n} dR_o \quad K \le R_o \le X \tag{20}$$

$$-U_z = \int_{R_o}^1 -(R_o - \frac{X^2}{R_o})^{1/n} dR_o \quad X \le R_o \le 1 \tag{21}$$

Equations (20) and (21) are equal at $R_o = K$, then:

$$U_t = \int_K^X (\frac{X^2}{R_o} - R_o)^{1/n} dR_o - \int_X^1 (R_o - \frac{X^2}{R_o})^{1/n} dR_o \tag{22}$$

Combination of equations (8) and (12) at cylinder wall, where $U_z = -U_t$ and $V_z = -V_t$, yields:

$$V_t = U_t R_c \left[\frac{\left(\frac{K}{X}\right)^2 g(\rho_s - \rho_l) R_c}{2m} \right]^{\frac{1}{n}} \tag{23}$$

Equation (23) is a final relation between falling velocity of the cylinder and fluid and cylinder density difference via power-law model.

Determination of Rheological Parameters

The equation (23) can be also rearranged in the following form [14]:

$$\ln V_t = \frac{1}{n} \ln(\rho_s - \rho_l) + \ln \left\{ U_t R_c \left[\frac{\left(\frac{K}{X}\right)^2 g R_c}{2m} \right]^{\frac{1}{n}} \right\} \tag{24}$$

The consistency index (m) and power law index (n) of the solutions can be determined using the falling-cylinder rheometer in following steps:

1. For a power-law fluid a plot of $\ln V_t$ vs. $\ln(\rho_s - \rho_l)$ is a straight line with a slope equal to the power law index (n).
2. From equation (17), value of (X) is determined using n and K.
3. From equation (23), the value of U_t is calculated.
3. Using equation (24) with any pair of V_t and $(\rho_s - \rho_l)$, value of m is determined.

Experimental

The falling-cylinder rheometer consists of a long cylindrical container filled with a non-Newtonian fluid, in which the different cylinders with predesigned densities are allowed to fall under the influence of gravity (Figure 1). The axis of the falling cylinder is parallel to the gravity vector. The cylinders were made of cylindrical glass tubes and the ends of the cylinders were rounded to be hemispherical to ensure streamline flow and to reduce the entrance-and end effects. The dimensions of the cylinders used in the present investigation are given in Table 1. Some aluminium powder can be placed in the hollow cylinder to increase its effective density. This also stabilizes the motion of the cylinder which may deviate from the centerline. Also, a guide ring assembly at the top of the centerline was used to facilitate the releasing of the cylinder along the centerline of the filled cylindrical container.

The temperature of the fluid was controlled using the circulating fluid in the jacket of the container. The terminal velocity of the cylinder was determined by measuring the travelling time of the cylinder between two measurement lines inscribed on the cylindrical container. At the bottom of the container an exit valve was used to bring out the cylinder and to drain the test fluids.

The important parameters are the falling time and distance, cylinder and fluid densities, falling stability, cylinder eccentricity and temperature variations. Great care was taken to get accurate results. At the present investigation, the rheological properties of water solution of polyvinyl alcohol (MW=72000) at different concentrations (2 to 7 weight percents) were measured (Table 1). Due to wide range of viscosities encountered, the measurements were carried out on a number of falling cylinders of different geometries and densities. The cylinders were selected so as to slowly fall in order to have small errors in the total falling time as well as small inertia term in the governing motion equations. Therefore, for low viscosity liquids a low mass falling cylinder was used to ensure the terminal velocity is established before the cylinder reaches the bottom of the container. For each concentration of polyvinyl alcohol, a number of repeated measurements were made to ascertain the uncertainty in the experimental results. The accuracy of the apparatus operation was confirmed with viscosity measurement of distilled water.

Table 1. Dimensions of the cylinders and rheological properties of PVA solutions

Weight percent of PVA	$K = \frac{R_c}{R_t}$	***T(°C)***	***Fluid density*** $(\frac{gr}{cm^3})$	***Consistency index(m)*** $(\frac{N.\sec^n}{m^2})$	***Power law index (n)***	***Power law equation***	***L (cm)***
2	0.886	25	1.010	0.00494	0.790	$\tau = 4.94\times10^{-3}\dot{\gamma}^{0.790}$	15
3	0.886	25	1.015	0.00925	0.764	$\tau = 9.27\times10^{-3}\dot{\gamma}^{0.764}$	15
4	0.613	25	1.020	0.01557	0.734	$\tau = 1.56\times10^{-2}\dot{\gamma}^{0.734}$	10
5	0.568	25	1.024	0.02170	0.718	$\tau = 2.17\times10^{-2}\dot{\gamma}^{0.718}$	10
6	0.460	25	1.030	0.02616	0.691	$\tau = 2.616\times10^{-2}\dot{\gamma}^{0.691}$	10
7	0.460	25	1.035	0.03033	0.663	$\tau = 3.033\times10^{-2}\dot{\gamma}^{0.663}$	10

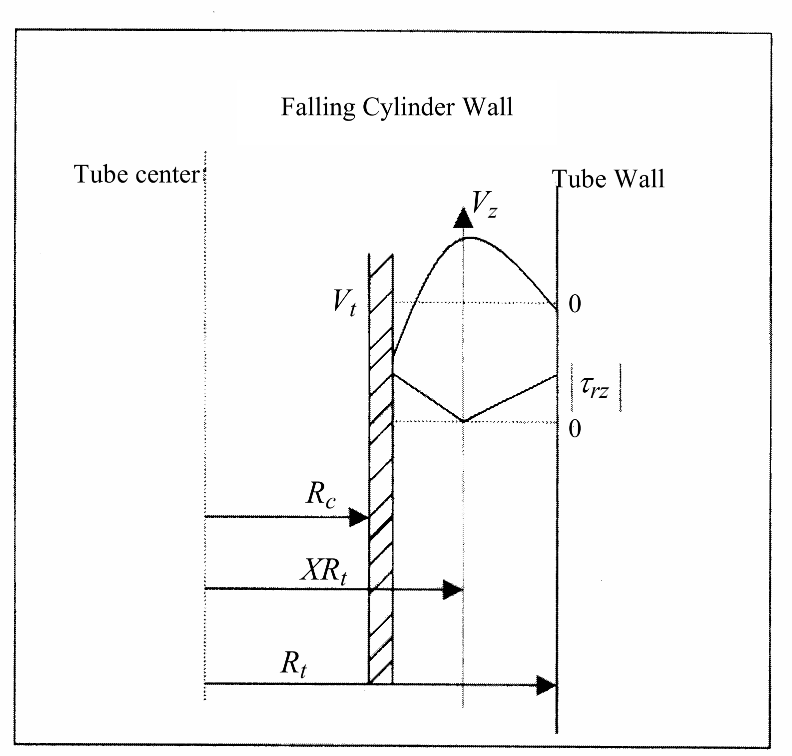

Figure 1. Symmetric diagram of falling cylinder rheometer

Results and Discussion

Table 1 shows the results of a number of cylinder runs using water solutions of polyvinyl alcohol at concentrations of 2 to 7 weight percents. The rheological properties of the solutions were calculated using the experimental data and equations 1 to 5 obtained by Etemad, et al. [14]. For several cases, the data obtained by falling cylinder rheometer were compared to the results of a Haake RV12 rotating rheometer which indicate very good agreement. Figures 3 and 4 present the experimental data of shear stress vs. shear rate and illustrate how the power law non-Newtonian model fits the experimental results of the present study. From the figures, increasing shear rate enhances shear stress but the slope decreases, which corresponds to pseudoplastic behavior. Figure 4 shows the results of Figure 3 on logarithmic coordinates and the linear relation between the shear stress and shear rate emphasizes that the rheological behaviour of the solutions obey the power-law model. The power-law and consistency indices can be obtained from the slopes and intersects of the straight lines, respectively.

Figure 5 illustrates the apparent viscosity vs. shear rate which calculated from the data of Figure 4. From Figure 5, increasing shear rate reductes the apparent viscosities of the solutions, which ultimately approach a constant value. This behavior is related to pseudoplasticity of the PVA solutions, which means that at high shear rates the apparent viscosity becomes constant and the flow behavior corresponds to that of a Newtonian fluid. Figure 6 shows the power-law index vs. concentration of the fluid. As seen in Figure 6, a straight line (equation 25) fits the data and as concentration of the fluid increases, the power law index decreases. The following equation is applicable to the experimental data of Figure 6.

$$n = -0.0249x + 0.8385 \quad (25)$$

Figure 7 indicates the effect of fluid concentration on the consistency index of the Polyvinyl alcohol solutions. Based on the results of Figure 7, the consistency index increases by increasing the concentration of the fluid. The following straight line relation between *m* and x was determined.

$$m = 0.0053x - 0.0056$$

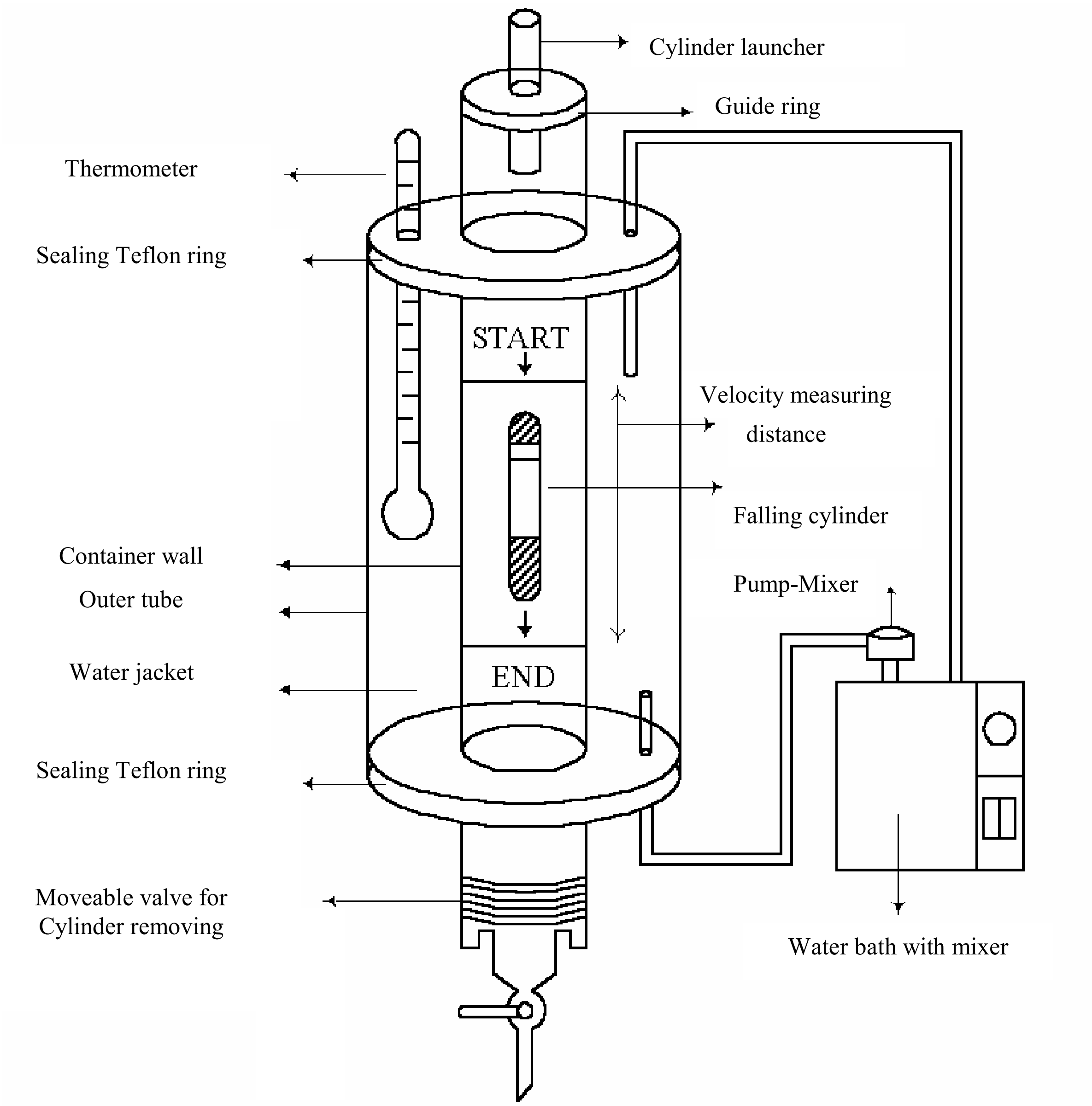

Figure 2. Schematic of falling- cylinder rheometer

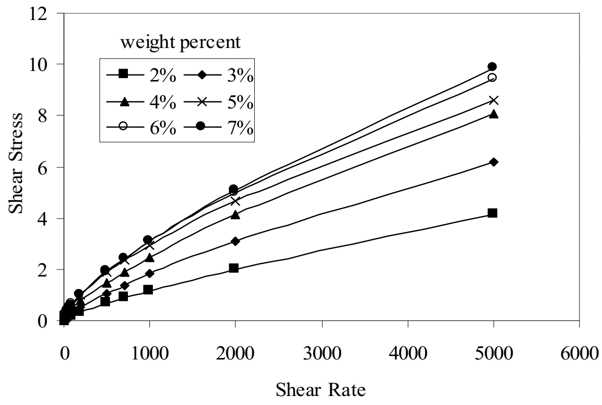

Figure 3. Shear stress vs. shear rate for various concentrations o f PVA solution

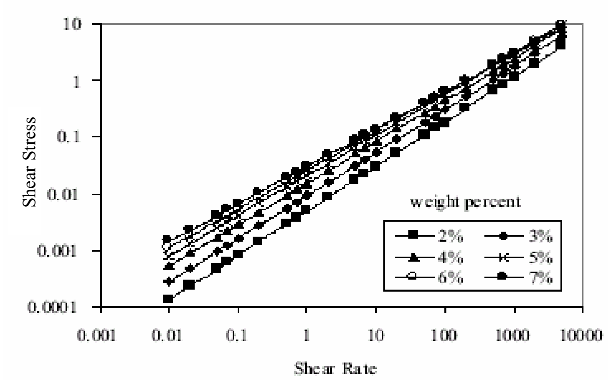

Figure 4. Shear stress vs. shear rate for various concentrations of PVA solution

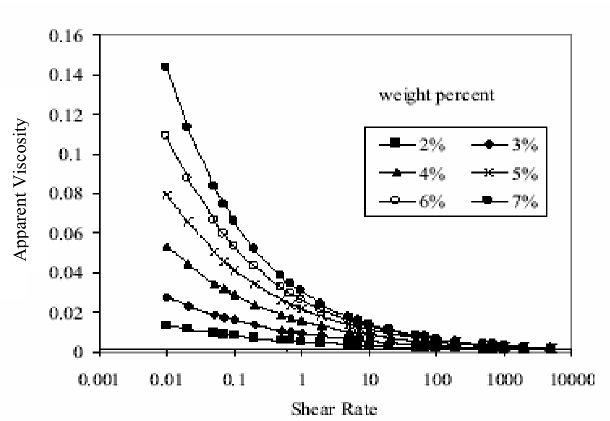

Figure 5. Apparent viscosity vs. shear rate for various PVA concentrations

Concluding Remarks

In the present investigation, an analytical relation (equation 24) was obtained that relates the rheological properties of the solution, the cylinder terminal velocity, and the density difference of the solution and cylinder. Then, a falling-cylinder rheometer (FCR) was designed and constructed, which uses hollow cylindrical glass tubes with hemispherical ends that are partially filled with aluminum powder to vary their effective densities. The terminal velocity of each cylinder was determined in two consecutive sections of the falling distances to ensure establishment of constant velocity. Using the experimental data obtained by the constructed FCR and employing the analytical results (equation 24), the rheological properties of the power-law model non-Newtonian fluids can be determined. As a specific case, the experimental data of distilled water was measured, which shows a very good agreement with available data in the literature. The rheological properties of the solutions of polyvinyl alcohol solutions with different concentrations were determined by the constructed rheometer, which indicate the excellent fit of the experimental results with power law model. Linear correlations were obtained between the power-law index as well as consistency index and concentration of the polyvinyl alcohol solutions.

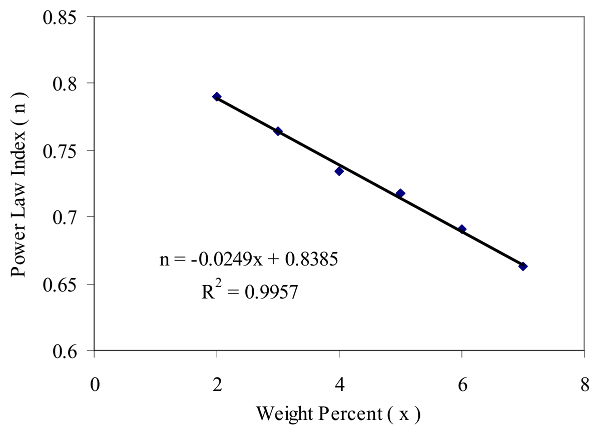

Figure 6. Power law index vs. P.V.A. Concentration

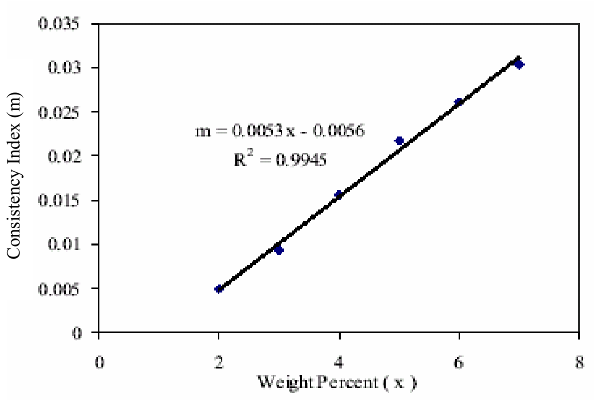

Figure 7. Consistency index vs. PVA concentration

Nomenclature

g	Gravitational acceleration, $\frac{m}{\sec^2}$
K	$\frac{R_c}{R_t}$, dimensionless
L	Length of cylinder, *m*
m	Consistency index, $\frac{N.\sec^n}{m^2}$
n	Power-law index, dimensionless
r	Distance from container center, m
R_c	Cylinder radius, m
R_o	$\frac{r}{R_t}$, dimensionless
R_t	Container radius, m
U_t	Dimensionless terminal velocity
U_z	Dimensionless vertical velocity
V_t	Terminal velocity, $\frac{m}{\sec}$
V_z	Vertical velocity, $\frac{m}{\sec}$
x	Weight percent of polyvinyl alcohol, dimensionless
X	$\frac{R_m}{R_t}$, dimensionless distance
ρ_s	Cylinder density, $\frac{kg}{m^3}$
ρ_l	Liquid density, $\frac{kg}{m^3}$
Δp	Pressure drop, $\frac{N}{m^2}$
τ_{rz}	Shear stress, $N.m^{-2}$
τ_w	Wall shear stress, $N.m^{-2}$

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